# University of Bahrain

*College of Information Technology*
*Department of Computer Science*

ITCS253 Discrete Structures II

Second Semester 2014/2015

Exam #1 – 75 Minutes

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|---|---|
| STUDENT NAME | |
| STUDENT# | |
| SECTION | |
| SERIAL | |

This exam contains **6** pages (including this cover page) and **6** questions. Check to see if any pages are missing. Enter all requested information.

**You are not allowed to use books, notes, or mobiles**

| Question | Points | Score |
|---|---|---|
| 1 | 7 | 6 |
| 2 | 7 | 7 |
| 3 | 8 | 5 |
| 4 | 4 | 4 |
| 5 | 7 | 7 |
| 6 | 7 | 7 |
| Total: | 40 | 36 |

**Instructor:** Dr. Ali Alsaffar   Sections# 1 & 2

## Answer all questions

**(1)** Let $f : \mathbf{A} \to \mathbf{R}$ defined by $f(x) = \dfrac{\sqrt{x-1}}{x-2}$ and $g : \mathbf{R} \to \mathbf{R}$ defined by $g(x) = |x| - 1$.

(a) [**2 points**] Find an appropriate domain **A** for $f$. Justify your answer.

In domain of A it's must $X \geq 3$ because it's impossible to the value under the root be negative and also it's not exist the $X = 2$ because the dominator will be zero so the domain will be like that

domain $= \{ \sqrt{2}, \frac{\sqrt{3}}{2}, \frac{2}{3}, \frac{\sqrt{3}}{4}, \ldots \}$

2

(b) [**3 points**] Find the range of $g$.

$$g(x) = \begin{cases} X \cancel{+} 1 & X \geq 0 \\ -X \cancel{+} 1 & X < 0 \end{cases}$$

In case : $X \geq 0$

let $y = X \cancel{+} 1$, solve for $X$

$X = y + 1$

$y + 1 \geq 0 \Rightarrow y \geq -1$

Range $= \{ y \in R \mid y \geq 1 \}$

2.5

In Case 2: $X < 0$

let $y = -X + 1$, solve for $X$

$-X = y - 1 \Rightarrow X = -y + 1$ $< 0 \Rightarrow y > 1$

$\cancel{X <}$ $-y + 1 < 0 \Rightarrow -y < -1 \Rightarrow y < 1$

Range $= \{ y \in R \mid y < 1 \}$

(c) [**2 points**] (i) Find $g \circ f$ (ii) Does $f \circ g$ exist? Justify your answer.

~~(i) $(g \circ f)(x) = g(f(x)) = g x$~~

(i) ~~$(g \circ f)$~~ In case : $x \geq 0$

$(g \circ f)(X) = g(f(x)) = g\left(\frac{\sqrt{X-1}}{X-2}\right) = \left|\frac{\sqrt{X-1}}{X-2}\right| - 1$

In case : $X < 0$

$(g \circ f)(X) = g(f(x)) = g\left(\frac{\sqrt{X-1}}{X-2}\right) = \frac{-\sqrt{X-1}}{X-2} - 1$

1

(ii) Doesn't exist because, not all element in $g(x)$ ~~is~~ in $f(x)$

1/2

**(2)** Define a function $f : \mathbf{R} \to \mathbf{R}$ by $f(x) = 3x^2 + 12x + 2$.

(a) [**4 points**] Find the range of $f$.

let $y = 3x^2 + 12x + 2$ , solve for $x$

$3x^2 + 12x + 2 - y = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 3$ , $b = 12$ , $c = 2 - y$

$$x = \frac{-12 \pm \sqrt{(12)^2 - 4(3)(2-y)}}{2(3)}$$

4

$$= \frac{-12 \pm \sqrt{144 - (12)(2-y)}}{6}$$

$$= \frac{-12 \pm \sqrt{144 - 24 + 12y}}{6} = \frac{-12 \pm \sqrt{120 + 12y}}{6}$$

$120 + 12y \geq 0 \Rightarrow -12y \geq +120$

$\Rightarrow y \geq \frac{+120}{-12}$

$\Rightarrow y \geq -10$

$\therefore$ Range $= \{y \in \mathbf{R} \mid y \geq -10\}$

(b) [**3 points**] Is $f$ one-to-one ? Prove your answer.

let $x_1, x_2 \in R$ and assume $f(x_1) = f(x_2)$

$3x_1^2 + 12x_1 + \cancel{2} = 3x_2^2 + 12x_2 + \cancel{2}$ ✓

$3x_1^2 + 12x_1 = 3x_2^2 + 12x_2$

$3x_1^2 - 3x_2^2 = -12x_1 + 12x_2$

$3(x_1^2 - x_2^2) = -12(x_1 - x_2)$

$3(x_1 - x_2)(x_1 + x_2) = -12(x_1 - x_2)$

3

Case 1: $x_1 = x_2$ we are done (one-to-one) ✓

Case 2: $x_1 \neq x_2$

$3(x_1 - \cancel{x_2})(x_1 + x_2) = -12(x_1 - \cancel{x_2})$

$x_1 + x_2 = \frac{-12}{3} = -4$ ✓

assume $f(x_1) = f(x_2)$ but $x_1 \neq x_2$ for example

$x_1 = -8$ , $x_2 = 4$ , then $x_1 + x_2 = -4$ ✓

$-8 + 4 = -4$

but $x_1 \neq x_2$ ~~so f isn't one~~

$\therefore$ f isn't one-to-one function.

**(3)** Consider the operation * defined on $G = \{1, 2, 3, 4\}$ by the table.

| * | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 3 | 4 | 1 | 2 |
| 2 | 4 | 1 | 2 | 3 |
| 3 | 1 | 2 | 3 | 4 |
| 4 | 2 | 3 | 4 | 1 |

Assume that * is an <u>*associative*</u> operation on $G$. Please answer the following questions and justify your answers.

(a) **[5 points]** Show that $(G, *)$ is a group.

# It's closed with G?
1, 2, 3, 4 ∈ G (all enteries in matrix is element ∈ in G)
∴ It's closed with G

1

# Identity element?
left Identity element:
~~e * a =~~ let a, e ∈ G and a = 1 and e = 3 (from Table)
~~a~~ e * a = a ⇒ 3 * ~~1~~ = 1

1/2

Right Identity element:
let a, e ∈ G ~~e = 3~~ and a = 1, ~~e~~ = 3
a * e = a = ~~1~~ * 3 = 1
∴ There~~s~~ is identity element e = ?

(Note): Inverse in the ~~bac~~ back of Page

(b) **[1 point]** Show that * is commutative on $G$.

By looking to left diagnal we ~~s~~know the table is symmetric, so if ~~it's~~ it's is symmetric then it's commutative.

1

(c) **[2 points]** Let $x \in G$ and suppose $(2 * x)^{-1} = 3$. Find $x$.

$(2 * x)^{-1} = 3$

$(2 * x^{~~-1~~})^{-1} = x^{-1} * 2^{-1} = 3$ ✓ (Inverse of 2 is 4) ✓

$= x^{-1} * 4 = 3$ ✓

2

from table, the number how give us 3 ~~with~~ when we check is any number give ~~2~~ * with ~~2~~ ~~4~~ invers of 2 is 2

∴ $x^{-1} = 2$, by show the table we know the $x = 4$

# Inverse ?

let $a, a^{-1} \in G$ and $a = 1$ , $a^{-1} = 1$ ( from Table )

. left Inverse :

$a^{-1} * a = e \Rightarrow$ ~~1 3~~ $a^{-1} * a = 3 \Rightarrow 1 * 1 = 3$

1/2 . Right Inverse

$a * a^{-1} = e \Rightarrow a * a^{-1} = 3 \Rightarrow 1 * 1 = 3$

$1^{-1} = ?$

$2^{-1} = ?$

3 $\therefore$ It's has inverse

from previous information in question ~~we know~~ after ( associative )
and what we find ( closed with G , Identity element , Inverse )
G is group .

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~~Q3: Part c : complete the answer~~

~~$\therefore$ 3 is identity element of group G~~

$\therefore$ X

**(4)** **[4 points]** Suppose $G$ is a group. For any $a, b \in G$, if $a^{-2} = e$, show that

$$[(a * b)^{-1} * b]^2 = e$$

$[(a*b)^{-1}*b]^2 = [(a*b)^{-1}*b]*[(a*b)^{-1}*b]$ ✓

$= [b^{-1}*a^{-1}*b]*[b^{-1}*a^{-1}*b]$ ✓

$= [b^{-1}*a^{-1}*b*b^{-1}*a^{-1}*b]$

$= [b^{-1}*a^{-1}*\underbrace{(b*b^{-1})}_{e}*a^{-1}*b]$ ✓

4

$= [b^{-1}*\underbrace{(a^{-1}*e)}_{a^{-1}}*a^{-1}*b]$

$= [b^{-1}*\underbrace{(a^{-1}*a^{-1})}_{a^{-2}}*b]$ ✓

$= [b^{-1}*\underbrace{a^{-2}}_{e}*b]$ ✓

$= [(b^{-1}*e)*b] = [b^{-1}*b] = e$ ✓

**(5)** Answer the following questions.

(a) **[3 points]** Consider the sequence of numbers $3, 2, 1, 0, -1, -2, -3, -4, \ldots$.
Write a recurrence relation to the above sequence.

3 $a_0 =$ 3 ✓, $a_n =$ $a_0 - 1$ ✓, $n \geq$ ~~0~~ 1 ✓

(b) **[4 points]** Find a solution to the recurrence relation in (a) using the *Iteration Method.*

$a_1 = a_0 - 1$

$a_2 = a_1 - 1 = a_0 - 1 - 1 = a_0 - 2$

$a_3 = a_2 - 1 = a_0 - 1 - 1 - 1 = a_0 - 3$

$a_4 = a_3 - 1 = a_0 - 1 - 1 - 1 - 1 = a_0 - 4$ ✓

$\vdots$

4

$\cancel{a_n = a_0 - (n-1) = a_0 - n + 1}$

$\cancel{= 3 - n + 1}$

$a_n = a_0 - n$ ✓

$= 3 - n$